Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

NOTE ON THE GENERAL EQUATION OF THE SECOND DEGREE.

By GEORGE R. DEAN, Missouri School of Mines, Rolla, Mo.

This paper is written for the benefit of the average sophomore who gets very little benefit out of the general discussion given in most text-books, and in most cases is bewildered with details.

Let the directrix be $x\cos\alpha+y\sin\alpha-p=0$, focus (h, k), eccentricity e. Then the equation of the conic is

$$(1-e^2\cos^2\alpha)x^2-2e^2\sin\alpha\cos\alpha xy+(1-e^2\sin^2\alpha)y^2-2(h-e^2p\cos\alpha)x$$
$$+2(k-e^2p\sin\alpha)y+h^2+k^2-e^2p^2=0.$$

Comparing this with the equation

$$ax^2+2bxy+cy^2+2dx+2ey+f=0,$$

we have the six equations,

$$1-e^2\cos^2\alpha=an\ \ldots\ (1). \qquad (1-e^2\sin^2\alpha)=cn\ \ldots\ (2).$$
$$-2e^2\sin\alpha\cos\alpha=2bn\ \ldots\ (3). \qquad h-e^2p\cos\alpha=-dn\ \ldots\ (4).$$
$$k-e^2p\sin\alpha=-en\ \ldots\ (5). \qquad h^2+k^2-e^2p^2=fn\ \ldots\ (6).$$

Eliminating α between (1) and (2),

$$2-e^2=(a+c)n, \text{ or } n=\frac{2-e^2}{a+c}.\ \ldots\ (7).$$

Subtracting the square of (3) from the product of (1) and (2),

$$1-e^2=(ac-b^2)n^2, \text{ or } n^2=\frac{e^2-1}{b^2-ac}.\ \ldots\ (8).$$

Since n^2 is positive, e^2-1 and b^2-ac must have the same sign. Eliminating n between (7) and (8) and solving for e^2,

$$e^2=\frac{[(a+c)^2+4(b-ac)]\pm(a+c)\sqrt{(a+c)^2+4(b^2-ac)}}{2(b^2-ac)}.$$

The value of e^2 being positive, the negative value is rejected. Substituting in $n=\frac{2-e^2}{a+c}$, n is found.

From (1) and (2), $\cos 2\alpha=\dfrac{(c-a)n}{e^2}$.

From (3), $\sin 2\alpha=\dfrac{-2bn}{e^2}$.

These equations determine α without ambiguity. Substituting for α, e, n in the equations

$$h-e^2p\cos\alpha=-dn,$$

$$k-e^2p\sin\alpha=-en,$$

$$h^2+k^2-e^2p^2=fn,$$

and solving for h, k, p, the curve is completely determined.

The solution of these last equations will be much simpler if the given equation is first transformed to the center, for we will then have

$$h=e^2p\cos\alpha,\quad k=e^2p\sin\alpha,\quad h^2+k^2-e^2p^2=f'n,$$

f' being obtained by substituting the coördinates of the center in the left hand member of the given equation.

If a'=semi-major axis, b'=semi-minor axis, we have $a'=ep$, $b'=a'\sqrt{(1-e^2)}$.

Let us take the equation $3x^2+2xy+3y^2-16y+20=0$.

Transformed to centre $(-1, 3)$, $3x^2+2xy+3y^2-4=0$, we find

$$e^2=\tfrac{1}{2},\quad n=\frac{2-\frac{1}{2}}{6}=\tfrac{1}{4},\quad \sin 2\alpha=-1,\quad \cos 2\alpha=0.$$

$$\alpha=135^\circ,\quad h=-\frac{p}{\sqrt{8}},\quad k=\frac{p}{\sqrt{8}},\quad p=\pm2,\quad a'=\sqrt{2},\quad b'=1.$$

In general the equations for h, k, p give two values of each quantity showing that the conic has two directrices and two foci.

If $e=1$, then $h-p\cos\alpha=-dn$, $k=-p\sin\alpha=-en$, $h^2+k^2-p^2=fn$. The terms containing p^2 cancel, showing that the parabola has one directrix and one focus at infinity.

◆

INTEGRATION OF ELLIPTIC INTEGRALS.

By G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

[Continued from April Number.]

$$\therefore\ B_0=\frac{1}{\pi}\int_0^{2\pi}\frac{d\varphi}{(1+e^2-2e\cos\varphi)^{\frac{3}{2}}}$$